FST 6-6 Notes
Topic: Conditional Probability
GOAL
Introduce the language and notation of conditional probability and apply conditional probability to situations where the answers are not at all obvious.

SPUR Objectives

D Calculate probabilities using the definition of conditional probability.

I Calculate probabilities in real situations.

Vocabulary

conditional probability of an event, $P(B|A)$

Warm-up

Suppose 60% of the singers in a school play are in the school choir. In the school as a whole, suppose 10% of the students are in the choir and 5% are in the school play. Finally, suppose there are 600 students in the school.

a. If a student in the play is randomly chosen, what is the probability that the student is in the choir?

$$\text{Prob} = \frac{\text{\# students in play + choir}}{\text{\# students in play}} = 60\%$$

b. If a student in the choir is randomly chosen, what is the probability that student is in the play?

$$\text{Prob} = \frac{\text{\# students in play + choir}}{\text{\# students in choir}} = \boxed{\frac{18}{60}}$$

$10\%(600) = 60$ choir
$5\%(600) = 30$ play
$60\%(30) = 18$ play also in choir

c. If a student in the school is randomly chosen, what is the probability that the student is in both the play and the choir?

$$\text{Prob} = \frac{\text{\# of students in play + choir}}{\text{\# students in School}} = \boxed{\frac{18}{600}}$$

Conditional Probability

What is the probability that a random student in our school walked to school today?

Let W = walked
S = students

$$P(W) = \frac{N(W)}{N(S)} = \frac{\text{\# of walkers}}{\text{\# of students in school}}$$

What is the probability that a student who lives over 1 mile from school walked to school today?

Let W = walked
M = Live over 1 mile
P(walked given lived over 1 mile)

$$P(W|M) = \frac{P(W \cap M)}{P(M)}$$

Titanic Table 1 below lists the number of passengers and crew who survived and died (the possible outcomes) in the sinking of the Titanic, categorized by status (first-class, second-class, third-class, and crew).

Titanic Table 1: Status and Survival

	First	Second	Third	Crew	Total
Survived	203	118	178	212	711
Died	122	167	528	673	1490
Total	325	285	706	885	2201

Source: British Wreck Commissioner's Inquiry Report

1. What is the probability a passenger survived? S

$$P(S) = \frac{N(S)}{N(\text{Total})} = \frac{711}{2201}$$

2. What is the probability a passenger survived and was in second class?

Let A = passenger survived Let B = second class

Then $P(A \cap B)$ = the "intersection" of A and B, what A and B have in common.

Both

$$P(A \cap B) = \frac{N(A \cap B)}{N(\text{Total})} = \frac{118}{2201}$$

3. What is the probability a passenger survived given they were in second class?

$$P(\underset{A}{\text{Survived}} \text{ given } \underset{B}{2^{nd} \text{ class}}) = \frac{\overset{P(\text{Both})}{P(A \cap B)}}{\underset{P(2^{nd}\text{ class})}{P(B)}} = \frac{\frac{118}{2201}}{\frac{285}{2201}}$$

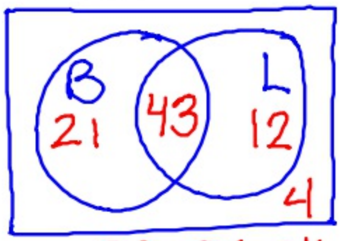
Definition of Conditional Probability

The **conditional probability of an event** B given an event A, written $P(B \mid A)$, is $\frac{P(A \cap B)}{P(A)}$.

$$\frac{118}{2201} \div \frac{285}{2201}$$

$$\frac{118}{\cancel{2201}} \cdot \frac{\cancel{2201}}{285} = \boxed{\frac{118}{285}}$$

Example 1: Let B = a person eats a good breakfast; Let L = a person eats a good lunch. Suppose in a group of 80 people, 43 eat good breakfasts and good lunches, 21 eat a good breakfast but not a good lunch, 12 eat a good lunch but not a good breakfast, and the rest eat neither a good lunch nor a good breakfast.



$80 - 76 = 4$

a. Find $P(B \cap L)$

Both

$$P(B \cap L) = \frac{N(B \cap L)}{N(\text{Total})} = \frac{43}{80}$$

b. Find $P(L \mid B)$

Both

$$P(\text{Lunch given breakfast}) = \frac{P(L \cap B)}{P(B)} = \frac{\frac{43}{80}}{\frac{64}{80}} = \boxed{\frac{43}{64}}$$

21+43

c. Find $P(B \mid L)$

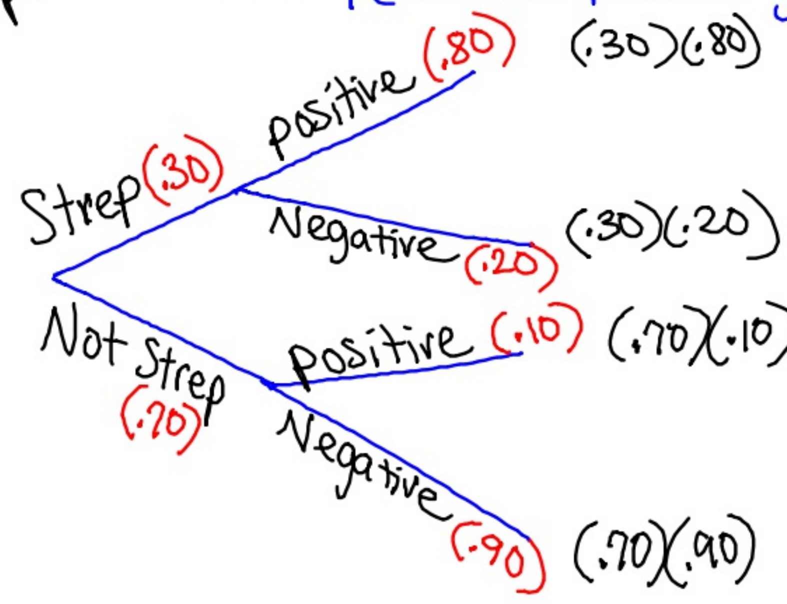
Both

$$P(\text{Breakfast given Lunch}) = \frac{P(L \cap B)}{P(L)} = \frac{\frac{43}{80}}{\frac{55}{80}} = \boxed{\frac{43}{55}}$$

43+12

Example 2: An article in the Journal of the American Medical Association in 1997 reported that, when people go to their doctor's office with a sore throat and think they might have strep throat, 30% actually have strep throat. It noted that a current test for strep throat was 80% accurate if you have strep throat and 90% accurate if you do not. What is the probability that a person who receives a positive result from this test does not have the disease? P(Not Strep throat given tested positive)

False positive



Not strep + positive

$$= \frac{P(\text{Both})}{P(\text{positive})}$$

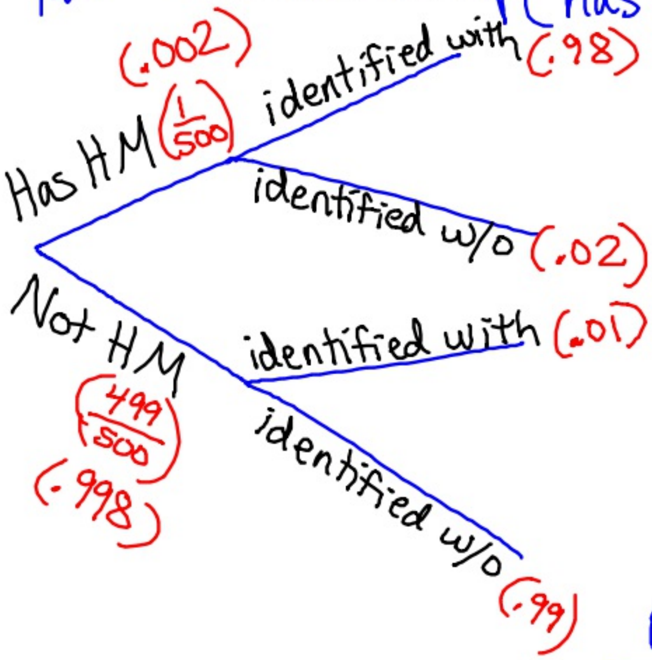
$$= \frac{(.70)(.10)}{(.30)(.80) + (.70)(.10)}$$

$$= \frac{.07}{.31} = 22.6\%$$

HM = Hazardous Material

Example 3: Suppose that 1 in 500 airline passengers carry some hazardous material on them when on a plane. Further suppose that an airport screening device accurately identifies 98% of people with hazardous materials that pass through it, and accurately identifies 99% of people without hazardous materials. If a person is identified by the machine as having hazardous materials, what is the probability that the person actually has these kinds of materials?

P(has HM given identified with)



	Has HM .002	Not HM .998
identified with	.98 (.002)(.98)	.01 (.01)(.998)
identified w/out	.02 (.02)(.002)	.99 (.99)(.998)

$$\frac{P(\text{Both})}{P(\text{identified with})} = \frac{(.002)(.98)}{(.002)(.98)+(.01)(.998)}$$

$$= \frac{.00196}{.01194} = \boxed{16.4\%}$$